Math 70 7.4, 7.5-1st

Lesson 22

7.4 Adding and Subtracting Rational Expressions which do not have a common denominator

7.5 Simplifying complex rational expressions

Objectives

1) Find common denominators

2) Rewrite equivalent fractions with common denominators
   ⇒ "write in higher terms"

3) Add or subtract rational expressions

4) Simplify complex rational expressions.

Math 70 **7.4 Adding and Subtracting Rational Expressions**
7.5 Simplifying Complex Rational Expressions

Objectives

1) Find common denominators
2) Rewrite equivalent fractions with common denominators (write in "higher terms", opposite of "simplify")
3) Add or subtract rational expressions
4) Simplify complex rational expressions
   a. Method 1: Use order of operations
   b. Method 2: Multiply entire fraction by $\dfrac{\frac{LCD}{1}}{\frac{LCD}{1}}$

Examples

Perform the indicated operations, then fully simplify if possible.

1) $\dfrac{x}{x-6}+\dfrac{6}{6-x}$

2) $\dfrac{4x-1}{4x^2+8x+4}-\dfrac{2x-3}{2x^2-2x-4}$

3) $\dfrac{b}{b^2-25}-\dfrac{5}{b+5}+\dfrac{6}{b}$

4) $\left(\dfrac{x}{x+3}-\dfrac{x}{x-3}\right)\div\dfrac{x}{7x+21}$

Simplify

5) $\dfrac{\dfrac{2}{x+5}+\dfrac{6}{x+7}}{\dfrac{2x+11}{x^2+12x+35}}$

6) $\dfrac{\dfrac{14}{15-x}+\dfrac{15}{x-15}}{\dfrac{8}{x}+\dfrac{7}{x-15}}$

Bonus Challenge problems

7) $4x^{-2}+x^{-3}(5x+2)$

8) $\dfrac{-y^3z+2y^2z^2-15yz^3}{z^2y-2y^2z}+\dfrac{y^3z-2y^2z^2-8yz^3}{2y^3z+3y^2z^2-2yz^3}\div\dfrac{16z^2-y^2}{y^3+64z^3}$

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Perform the indicated operations and fully simplify.

① $\dfrac{x}{x-6} + \dfrac{6}{6-x}$

↑

addition and subtraction of fractions require common denominator

$x-6 \neq 6-x$

ex. if $x=2$ $\quad x-6 \rightarrow 2-6=-4$

but $\quad 6-x \Rightarrow 6-2=4$

$-4 \neq 4$

Factor $6-x$

$= -x+6$ standard form

$= -(x-6)$ GCF -1

$= \dfrac{x}{x-6} + \dfrac{6}{-(x-6)}$

move negative to numerator since $\dfrac{6}{-1} = \dfrac{-6}{1}$

$= \dfrac{x}{x-6} + \dfrac{-6}{x-6}$

$= \dfrac{x+(-6)}{x-6}$

add numerators
Keep common denominator

$= \dfrac{x-6}{x-6}$

cancel common factors

$= \boxed{1}$

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Perform the indicated operations and fully simplify.

(2) $\dfrac{4x-1}{4x^2+8x+4} - \dfrac{2x-3}{2x^2-2x-4}$

$\uparrow$ subtract requires common denominator

Factor denominators completely

$A \Rightarrow 4x^2+8x+4$ GCF $= 4$

$= 4(x^2+2x+1)$ trinomial

$= 4(x+1)(x+1)$

$= 4(x+1)^2$

$B \Rightarrow 2x^2-2x-4$ GCF $= 2$

$= 2(x^2-x-2)$ trinomial

$= 2(x-2)(x+1)$

Construct LOWEST common denom = Least Common Multiple

LCM of 4 and 2 $\Rightarrow 4$

LCM of $(x+1)^2$ and $(x+1) \Rightarrow (x+1)^2$

LCM of 1 and $(x-2) \Rightarrow (x-2)$

LCD $= 4(x+1)^2(x-2)$

$$= \frac{4x-1}{4(x+1)^2} - \frac{2x-3}{2(x-2)(x+1)}$$

Multiply each fraction by missing factors numerator and denominator to create LCD.

$$= \frac{(4x-1)(x-2)}{4(x+1)^2(x-2)} - \frac{2(2x-3)(x+1)}{4(x+1)^2(x-2)}$$

Simplify numerators so like terms can be combined

$$= \frac{4x^2-9x+2 - 2(2x^2-x-3)}{4(x+1)^2(x-2)}$$ FOIL twice

$$= \frac{4x^2-9x+2 - 4x^2+2x+6}{4(x+1)^2(x-2)}$$ distribute -2

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(2) con't.

$$= \frac{-7x+8}{4(x+1)^2(x-2)}$$ combine like terms

(3) $$\frac{b}{b^2-25} - \frac{5}{b+5} + \frac{6}{b}$$

order of operations : add and subtract from left to right

factor completely

$$= \frac{b}{(b+5)(b-5)} - \frac{5}{(b+5)} + \frac{6}{b}$$

LCD for all 3 fractions $b(b+5)(b-5)$

multiply by missing factors to write equivalent fractions

$$= \frac{b \cdot b}{b(b+5)(b-5)} - \frac{5 \cdot b \cdot (b-5)}{b(b+5)(b-5)} + \frac{6(b+5)(b-5)}{b(b+5)(b-5)}$$

Simplify numerators so like terms can be combined

$$= \frac{b^2 - 5b^2 + 25b + 6(b^2-25)}{b(b+5)(b-5)}$$

$$= \frac{b^2 - 5b^2 + 25b + 6b^2 - 150}{b(b+5)(b-5)}$$

$$= \boxed{\frac{2b^2 + 25b - 150}{b(b+5)(b-5)}}$$

trinomial, leading coef $\neq 1$

$2(-150)$

-300

25

$-1, 300$

$-2, 150$

$-3, 100$

$-4, 75$

$-5, 60$

$-6, 50 \longrightarrow 44$

$-10, 30 \longrightarrow 20$

prime

④ $\left(\frac{x}{x+3}-\frac{x}{x-3}\right)\div\frac{x}{7x+21}$

$= \left(\frac{x}{x+3} - \frac{x}{x-3}\right)\cdot\frac{7x+21}{x}$ LCD $= (x+3)(x-3)$

$= \left(\frac{x}{x+3}\cdot\frac{x-3}{x-3} - \frac{x}{x-3}\cdot\frac{x+3}{x+3}\right)\cdot\frac{7(x+3)}{x}$

$= \left(\frac{x^2-3x-(x^2+3x)}{(x-3)(x+3)}\right)\cdot\frac{7(x+3)}{x}$

$= \frac{x^2-3x-x^2-3x}{(x-3)(x+3)}\cdot\frac{7(x+3)}{x}$

$= \frac{-6x}{(x-3)\cancel{(x+3)}}\cdot\frac{7\cancel{(x+3)}}{x}$

$= \boxed{\frac{-42}{x-3}}$ $(x\neq 0,-3)$

$\frac{x}{x}$ cancels $\Rightarrow x\neq 0$

$\frac{x+3}{x+3}$ cancels $\Rightarrow x\neq -3$

Math 70 Simplifying Complex Fractions

Simplify a complex fraction means rewrite so that there are no fractions-within-fractions remaining. A fraction-within-a-fraction is never a simplified answer.

Simplify.

(5) $$\frac{\frac{2}{x+5}+\frac{6}{x+7}}{\frac{2x+11}{x^2+12x+35}}$$

Method 1: find the LCD of all denominators within the larger fraction, multiply by $\frac{\left(\frac{LCD}{1}\right)}{\left(\frac{LCD}{1}\right)}=1$

denominators: $(x+5)$, $(x+7)$, $x^2+12x+35=(x+5)(x+7)$

$LCD=(x+5)(x+7)$

$$=\frac{\left(\frac{2}{x+5}+\frac{6}{x+7}\right)\frac{(x+5)(x+7)}{1}}{\left(\frac{2x+11}{\cancel{(x+5)}\cancel{(x+7)}}\right)\frac{\cancel{(x+5)}\cancel{(x+7)}}{1}}$$

← notice that distributing is needed before canceling because denom $(x+5)$ cancels differently from denominator $(x+7)$

$$=\frac{\frac{2}{\cancel{(x+5)}}\cdot\cancel{(x+5)}(x+7)+\frac{6(x+5)\cancel{(x+7)}}{\cancel{(x+7)}}}{2x+11}$$

$$=\frac{2(x+7)+6(x+5)}{2x+11}$$ ← use parentheses

$$=\frac{2x+14+6x+30}{2x+11}$$ ← distribute

$$=\frac{8x+44}{2x+11}=\frac{4\cancel{(2x+11)}}{\cancel{(2x+11)}}=\boxed{4}$$ ← factor + cancel to simplify

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(5) Method 2: Add the two fractions in numerator, then divide result.

$$\frac{\left(\frac{2}{x+5}+\frac{6}{x+7}\right)}{\left(\frac{2x+11}{(x+5)(x+7)}\right)}$$

$$=\frac{\left(\frac{2(x+7)}{(x+5)(x+7)}+\frac{6(x+5)}{(x+5)(x+7)}\right)}{\left(\frac{2x+11}{(x+5)(x+7)}\right)}$$ ← find common denom to add fractions

$$=\frac{\left(\frac{2x+14+6x+30}{(x+5)(x+7)}\right)}{\left(\frac{2x+11}{(x+5)(x+7)}\right)}$$

$$=\frac{\left(\frac{8x+44}{(x+5)(x+7)}\right)}{\left(\frac{2x+11}{(x+5)(x+7)}\right)}$$ ← rewrite this fraction bar using ÷ symbol

$$=\frac{8x+44}{(x+5)(x+7)}\div\frac{2x+11}{(x+5)(x+7)}$$

$$=\frac{8x+44}{\cancel{(x+5)}\cancel{(x+7)}}\cdot\frac{\cancel{(x+5)}\cancel{(x+7)}}{2x+11}$$ ← multiply by reciprocal

$$=\frac{8x+44}{2x+11}$$

$$=\frac{4\cancel{(2x+11)}}{\cancel{(2x+11)}}$$ ← factor and cancel

$$=\boxed{4}$$

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6 $$\frac{\frac{14}{15-x}+\frac{15}{x-15}}{\frac{8}{x}+\frac{7}{x-15}}$$

Method 1 multiply by LCD

$$\text{denom}\begin{cases}(15-x) = -(x-15)\\ (x-15)\\ x\\ (x-15)\end{cases}$$

$LCD = x(x-15)$ if we move one negative to its numerator

$$= \frac{\left(\frac{-14}{x-15}+\frac{15}{x-15}\right)\frac{x(x-15)}{1}}{\left(\frac{8}{x}+\frac{7}{x-15}\right)\frac{x(x-15)}{1}}$$ ← dist ← dist

$$= \frac{\frac{-14x\cancel{(x-15)}}{\cancel{(x-15)}}+\frac{15\cdot x\cancel{(x-15)}}{\cancel{(x-15)}}}{\frac{8}{\cancel{x}}\cdot\cancel{x}(x-15)+\frac{7}{\cancel{(x-15)}}x\cancel{(x-15)}}$$ ← cancels differently

$$= \frac{-14x+15x}{8(x-15)+7x}$$

$$= \frac{x}{8x-120+7x}$$ ← dist

$$= \frac{x}{15x-120}$$ ← combine

$$= \boxed{\frac{x}{15(x-8)}}$$

leave result in factored form because it's a rational

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(6) Method 2: add numerators, add denominators, divide

$$\frac{14}{15-x}+\frac{15}{x-15}$$

$$=\frac{-14}{x-15}+\frac{15}{x-15}$$

$$=\frac{1}{x-15}$$

add numerators

$$\frac{8}{x}+\frac{7}{x-15}$$

$$=\frac{8(x-15)+7x}{x(x-15)}$$

$$=\frac{8x-120+7x}{x(x-15)}$$

$$=\frac{15x-120}{x(x-15)}$$

$$=\frac{15(x-8)}{x(x-15)}$$

add denominators

$$\frac{\frac{1}{(x-15)}}{\frac{15(x-8)}{x(x-15)}}$$

write with ÷ symbol

$$=\frac{1}{x-15}\div\frac{15(x-8)}{x(x-15)}$$

mult by reciprocal

$$=\frac{1}{\cancel{(x-15)}}\cdot\frac{x\,\cancel{(x-15)}}{15(x-8)}$$

$$=\boxed{\frac{x}{15(x-8)}}$$

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7 $4x^{-2} + x^{-3}(5x+2)$

$= \frac{4}{x^2} + \frac{5x+2}{x^3}$ LCD $= x^3$

$= \frac{4}{x^2} \cdot \frac{x}{x} + \frac{5x+2}{x^3}$

$= \frac{4x + 5x + 2}{x^3}$

$= \boxed{\frac{9x+2}{x^3}}$

$4x^{-2}$ no parentheses so exp -2 belongs to x only

Not $\frac{1}{4x^2} = (4x^2)^{-1}$

Not $\frac{1}{16x^2} = (4x)^{-2}$

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(8) Simplify.

$$\frac{-y^3z+2y^2z^2-15yz^3}{z^2y-2y^2z} \underset{\uparrow \text{ add}}{+} \frac{y^3z-2y^2z^2-8yz^3}{2y^3z+3y^2z^2-2yz^3} \underset{\uparrow \text{ divide}}{\div} \frac{16z^2-y^2}{y^3+64z^3}$$

Order of operations: divide before add. Mult by reciprocal:

$$= \frac{-y^3z+2y^2z^2-15yz^3}{z^2y-2y^2z} + \underbrace{\frac{y^3z-2y^2z^2-8yz^3}{2y^3z+3y^2z^2-2yz^3} \cdot \frac{y^3+64z^3}{16z^2-y^2}}_{\text{To multiply, factor each completely.}}$$

Factoring: $y^3z-2y^2z^2-8yz^3$

GCF $= yz(y^2-2yz-8z^2)$

trinomial leading coef 1

$-8 \leftarrow$ factors mult to -8

$-4 \quad 2$

$-2 \leftarrow$ factors add to -2

$$= \underline{\underline{yz(y-4z)(y+2z)}}$$

$2y^3z+3y^2z^2-2yz^3$

GCF $= yz(2y^2+3yz-2z^2)$

$2(-2) \leftarrow a \cdot c$ #s mult to -4

-4

$4 \quad -1$

$3 \leftarrow b$ #s add to 3

trinomial leading coef ≠ 1 "double X" = reminder to rewrite & group

$$= yz\{2y^2 + \underbrace{4yz - yz}_{\text{was } 3yz\text{, now rewritten using}} - 2z^2\}$$

grouping

$$= yz\{2y(y+2z) - z(y+2z)\}$$

$$= yz\{(y+2z)(2y-z)\}$$

$$= \underline{\underline{yz(y+2z)(2y-z)}}$$

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(B) con't

Factoring con't.

$y^3 + 64z^3$ sum of cubes

$(y)^3 \quad (4z)^3$

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

S O AP

$$= \underline{\underline{(y+4z)(y^2 - 4yz + 16z^2)}}$$

$16z^2 - y^2$ difference of squares

$$= \underline{\underline{(4z-y)(4z+y)}}$$

Rewrite w/ factoring:

$$= \frac{-y^3z + 2y^2z^2 - 15yz^3}{z^2y - 2y^2z} + \frac{\cancel{yz}(y-4z)\cancel{(y+2z)}}{\cancel{yz}\cancel{(y+2z)}(2y-z)} \cdot \frac{\cancel{(y+4z)}(y^2-4yz+16z^2)}{(4z-y)\cancel{(4z+y)}}$$

Divide out common factors. $y+4z = 4z+y$ can be cancelled

$4z - y = -y + 4z = -(y-4z)$

must factor out (-1) to cancel!

$$= \frac{-y^3z + 2y^2z^2 - 15yz^3}{z^2y - 2y^2z} - \frac{\cancel{(4z-y)}}{(2y-z)} \cdot \frac{(y^2-4yz+16z^2)}{\cancel{(4z-y)}}$$

$$= \frac{-y^3z + 2y^2z^2 - 15yz^3}{z^2y - 2y^2z} - \frac{y^2-4yz+16z^2}{2y-z}$$

To subtract, need a common denominator

Factor this denominator

$z^2y - 2y^2z$

GCF $= yz(z-2y)$

$2y - z \neq z - 2y$! Must factor out -1

(or mult: $\frac{-1}{-1} = +1$)

$$= \frac{-(-y^3z + 2y^2z^2 - 15yz^3)}{-yz(z-2y)} - \frac{y^2-4yz+16z^2}{2y-z}$$

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(B) cont

$$= \frac{y^3z - 2y^2z^2 + 15yz^3}{yz(2y-z)} - \frac{y^2 - 4yz + 16z^2}{(2y-z)}$$

* Astute observers might see a lot of yz in the first denominator and another method....

$$LCD = yz(2y-z)$$

$$= \frac{y^3z - 2y^2z^2 + 15yz^3}{yz(2y-z)} - \frac{yz(y^2 - 4yz + 16z^2)}{yz(2y-z)}$$

↑ dist $-yz$ to all terms in numerator

$$= \frac{\cancel{y^3z} - 2y^2z^2 + 15yz^3 - \cancel{y^3z} + 4y^2z^2 - 16yz^3}{yz(2y-z)}$$

combine like terms

$$= \frac{2y^2z^2 - yz^3}{yz(2y-z)}$$

← factor numerator

$$= \frac{yz^2(2y-z)}{yz(2y-z)}$$

divide out common factors

$$= \boxed{z}$$